On Stapp-Unruh-Mermin Discussion on Quantum Nonlocality:

Quantum Jumps and Relativity

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Abstract

We argue that the participants of the discussion have overlooked an essential circumstance, in view of which Stapp's fifth proposition fails. The circumstance is that though L and R measurements, being causally separated, are not invariantly time ordered, quantum-jump hypersurfaces associated with the measurements are causelikewise ordered. Stapp's fifth proposition is true iff L hypersurface precedes R one; but within the limits of special relativity, it is impossible to determine the causelike order of those hypersurfaces. The entire Stapp's construct is revised.

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Introduction

In a recent paper [1] Stapp made a sophisticated attempt to breathe new life into the problem of quantum nonlocality. The paper raised objections by Unruh [2] and Mermin [3], which resulted in a discussion on that problem [4-8].

The aim of Stapp's construct is the conclusion that there exists a backward-in-time influence. Stapp's fifth proposition is the cornerstone of the construct. Unruh and Mermin argue that the conclusion is untenable, though they grant the fifth proposition. Finkelstein [8] argues that the fifth proposition should be specified through consideration of a hypothetical world.

We argue that the participants of the discussion have overlooked an essential circumstance, in view of which Stapp's fifth proposition fails.

The circumstance is that though L and R measurements [1-8], being causally separated from each other, are not Lorentz-invariantly ordered in time, hypersurfaces of quantum jumps associated with the measurements are causelikewise ordered. Stapp's fifth proposition is true if and only if the L-hypersurface precedes the R-hypersurface. But within the limits of special relativity, it is impossible to determine— both theoretically and experimentally—what are those hypersurfaces and which is their causelike order.

Thus, the entire Stapp's construct should be revised, which is done in the present paper.

Invoking reference frames and time order for causally separated events is misleading, so that we use the geometric, or intrinsic approach to the problem.

1 Geometric, or intrinsic description of quantum field

1.1 Coordinate-free description

To avoid questions concerning reference frames, we shall be based on a geometric, or intrinsic description of a quantum field.

In the Heisenberg picture, a quantum field is

$$\phi_H \equiv \phi = \phi(p), \quad p \in M, \tag{1.1}$$

where M is the Minkowski spacetime,

$$\Psi_H \equiv \Psi \tag{1.2}$$

is a state vector;

$$\phi^{\text{class}}(p) = (\Psi, \phi(p)\Psi) \tag{1.3}$$

is a classical field.

1.2 Coordinate, or reference-frame description

Let $\{\tilde{x}\}$ and $\{\bar{x}\}$ be two Lorentzian coordinate systems,

$$p \leftrightarrow \tilde{x} \leftrightarrow \bar{x}, \qquad \bar{x} = \Lambda\tilde{x} + a. \tag{1.4}$$

We have

$$\phi(p) \leftrightarrow \{\tilde{\phi}_i(\tilde{x}) : i \in \mathcal{I}\} \leftrightarrow \{\bar{\phi}_i(\bar{x}) : i \in \mathcal{I}\}. \tag{1.5}$$

Ψ is the same both in the geometric and in all coordinate descriptions.

The transformation

$$\bar{\phi}_i^{\text{class}}(\bar{x}) = S_{ij}(\Lambda)\tilde{\phi}_j^{\text{class}}(\tilde{x}) \tag{1.6}$$

implies

$$\bar{\phi}_i(\bar{x}) = S_{ij}(\Lambda)\tilde{\phi}_j(\tilde{x}). \tag{1.7}$$

Now we have

$$S_{ij}(\Lambda)\tilde{\phi}_j(\tilde{x}) = U^{-1}(a,\Lambda)\tilde{\phi}_i(\tilde{x}')U(a,\Lambda) \tag{1.8}$$

where

$$\tilde{x} \leftrightarrow p \neq p' \leftrightarrow \tilde{x}' = \Lambda\tilde{x} + a = \bar{x}. \tag{1.9}$$

Let us take $\{\tilde{x}\}$ as a fiducial coordinate system and put

$$\tilde{\Psi} = \Psi, \tag{1.10}$$

then

$$(\tilde{\Psi}, \bar{\phi}_i(\bar{x})\tilde{\Psi}) = (\bar{\Psi}, \tilde{\phi}_i(\tilde{x}')\bar{\Psi}) \tag{1.11}$$

where

$$\bar{\Psi} = U(a,\Lambda)\tilde{\Psi}. \tag{1.12}$$

2 Quantum jumps: The problem of appropriate hypersurfaces and causelike ordering

In the Heisenberg picture, a state vector Ψ changes at and only at quantum jumps. To every quantum jump there corresponds a spacelike hypersurface. The hypersurfaces must be mutually disjoint: otherwise Ψ would be not defined. The quantum-jump hypersurfaces are causelikewise ordered:

$$\mathcal{S}_2 > \mathcal{S}_1, \qquad \text{or} \;\; \mathcal{S}_1 < \mathcal{S}_2. \tag{2.1}$$

Within the limits of special relativity, it is impossible to determine—both theoretically and experimentally—what are those hypersurfaces and which is, in the general case, their causelike order (see Section 4 below). It is special relativity that prevents a complete phenomenological mathematical description of quantum jumps and specifically measurements.

(A complete dynamical description of quantum jumps has been given in the series of our papers [9], which is beyond the scope of the present paper.)

3 Two-particle system

3.1 LR-system

Following [1] and the discussion, we consider a two-particle LR-system. Let

$$\rho_{LR} \equiv \rho \tag{3.1}$$

be a statistical operator of the system in the Heisenberg picture. We have

$$\rho_R = \mathrm{Tr}_L\rho, \quad \rho_L = \mathrm{Tr}_R\rho. \tag{3.2}$$

3.2 Local measurements and associated quantum jumps

A measurement implies instantaneity, which—in the limits of special relativity—makes sense locally only. A local measurement for L or R system results in an associated quantum jump for the state of LR system:

$$\{\mathcal{L}\} \Rightarrow \rho^0 \xrightarrow{\{\mathcal{L}\}} \rho^{\{\mathcal{L}\}}, \quad \{\mathcal{R}\} \Rightarrow \rho^0 \xrightarrow{\{\mathcal{R}\}} \rho^{\{\mathcal{R}\}} \tag{3.3}$$

where $\{\mathcal{L}\}$ stands for measuring an observable $\mathcal{L}$ of L system.

Let two measurements be performed: $\{\mathcal{L}\}$ at a point $p_{\{\mathcal{L}\}} \in M$ and $\{\mathcal{R}\}$ at a point $p_{\{\mathcal{R}\}} \in M$; let $\mathcal{S}_{\{\mathcal{L}\}}$ and $\mathcal{S}_{\{\mathcal{R}\}}$ be hypersurfaces of the associated quantum jumps. We write

$$p_{\{\mathcal{L}\}} < (>) p_{\{\mathcal{R}\}} \quad \text{iff} \quad \mathcal{S}_{\{\mathcal{L}\}} < (>) \mathcal{S}_{\{\mathcal{R}\}}. \tag{3.4}$$

Let $\mathcal{L}$ and $\mathcal{R}$ be integrals of motion. We have

$$\text{for} \quad p_{\{\mathcal{L}\}} < p_{\{\mathcal{R}\}}: \quad \rho^0 \xrightarrow{\{\mathcal{L}\}} \rho^{\{\mathcal{L}\}} \xrightarrow{\{\mathcal{R}\}} \rho^{\{\mathcal{L}\mathcal{R}\}}, \tag{3.5}$$

$$\text{for} \quad p_{\{\mathcal{R}\}} < p_{\{\mathcal{L}\}}: \quad \rho^0 \xrightarrow{\{\mathcal{R}\}} \rho^{\{\mathcal{R}\}} \xrightarrow{\{\mathcal{L}\}} \rho^{\{\mathcal{R}\mathcal{L}\}}; \tag{3.6}$$

the equality

$$\rho^{\{\mathcal{L}\mathcal{R}\}} = \rho^{\{\mathcal{R}\mathcal{L}\}} \tag{3.7}$$

holds since

$$[\mathcal{L}, \mathcal{R}] = 0. \tag{3.8}$$

Let $p_{\{\mathcal{L}\}}$ and $p_{\{\mathcal{R}\}}$ be causally separated, then there exist frames $\{\tilde{x}\}$ and $\{\bar{x}\}$, such that

$$\tilde{t}_{\{\mathcal{L}\}} < \tilde{t}_{\{\mathcal{R}\}}, \quad \bar{t}_{\{\mathcal{L}\}} > \bar{t}_{\{\mathcal{R}\}}; \tag{3.9}$$

thus, we must not appeal to the time order.

Within the limits of special relativity, it is impossible to determine which of the two cases (3.5),(3.6) takes place (see Section 4 below).

3.3 Nonselective and selective measurements

It is necessary to discriminate between nonselective and selective measurements [10]: in a (non)selective measurement the result is (not) registered.

Let

$$\mathcal{L} |l\rangle = l |l\rangle, \qquad \langle l | l\rangle = 1. \tag{3.10}$$

A nonselective measurement $[\mathcal{L}]$ is described by

$$\rho^0 \xrightarrow{[\mathcal{L}]} \rho^{[\mathcal{L}]} = \sum_l |l\rangle \langle l| \rho^0 |l\rangle \langle l|, \quad \rho^{[\mathcal{L}]} \neq \rho^0, \tag{3.11}$$

$$\rho_L^0 = \mathrm{Tr}_R \rho^0 \xrightarrow{[\mathcal{L}]} \rho_L^{[\mathcal{L}]} = \mathrm{Tr}_R \rho^{[\mathcal{L}]} = \sum_l |l\rangle \langle l| \rho_L^0 |l\rangle \langle l|, \quad \rho_L^{[\mathcal{L}]} \neq \rho_L^0, \tag{3.12}$$

$$\rho_R^0 = \mathrm{Tr}_L \rho^0 \xrightarrow{[\mathcal{L}]} \rho_R^{[\mathcal{L}]} = \mathrm{Tr}_L \rho^{[\mathcal{L}]} = \sum_l \langle l| \rho^0 |l\rangle = \mathrm{Tr}_L \rho^0, \quad \rho_R^{[\mathcal{L}]} = \rho_R^0. \tag{3.13}$$

A selective measurement (l) is defined by

$$\rho^0 \xrightarrow{(l)} \rho^{(l)} = \frac{|l\rangle \langle l| \rho^0 |l\rangle \langle l|}{\mathrm{Tr}\{|l\rangle \langle l| \rho^0 |l\rangle \langle l|\}} = \frac{|l\rangle \langle l| \rho^0 |l\rangle \langle l|}{\langle l| \rho^0_L |l\rangle}, \quad \rho^{(l)} \neq \rho^0, \tag{3.14}$$

$$\rho^0_L \xrightarrow{(l)} \rho^{(l)}_L = \mathrm{Tr}_R \rho^{(l)} = |l\rangle \langle l|, \quad \rho^{(l)}_L \neq \rho^0_L, \tag{3.15}$$

$$\rho^0_R \xrightarrow{(l)} \rho^{(l)}_R = \mathrm{Tr}_L \rho^{(l)} = \frac{\langle l| \rho^0 |l\rangle}{\langle l| \rho^0_L |l\rangle}, \quad \rho^{(l)}_R \neq \rho^0_R. \tag{3.16}$$

3.4 Pure initial state; symmetric case and reciprocity relation

Let

$$\rho^0 = |0\rangle \langle 0|. \tag{3.17}$$

A selective measurement (l) is described by

$$|0\rangle \xrightarrow{(l)} |(l)\rangle = \frac{|l\rangle \langle l|\, 0\rangle}{\|\, |l\rangle \langle l|\, 0\rangle \,\|} = \frac{|l\rangle \langle l|\, 0\rangle}{\|\, \langle l|\, 0\rangle \,\|}, \tag{3.18}$$

so that

$$|(l)\rangle = |l\rangle\, |R(l)\rangle, \quad |R(l)\rangle = \frac{\langle l|\, 0\rangle}{\|\, \langle l|\, 0\rangle \,\|}. \tag{3.19}$$

The symmetric case is that where

$$\frac{\langle R(l)|\, 0\rangle}{\|\, \langle R(l)|\, 0\rangle \,\|} = |l\rangle. \tag{3.20}$$

Let a selective measurement for L result in $|L\rangle$ and

$$\langle L|\, 0\rangle \propto |R\rangle, \tag{3.21}$$

so that

$$|L\rangle \Rightarrow |R\rangle. \tag{3.22}$$

From

$$\langle \perp R|\, R\rangle = 0 \tag{3.23}$$

it follows

$$\langle \perp R|\, \langle L|\, 0\rangle = 0, \quad \langle L|\, \langle \perp R|\, 0\rangle = 0, \quad \langle \perp R|\, 0\rangle \propto |\perp L\rangle, \tag{3.24}$$

so that

$$|\perp R\rangle \Rightarrow |\perp L\rangle. \tag{3.25}$$

Thus we have a reciprocity relation:

$$(|L\rangle \Rightarrow |R\rangle) \Rightarrow (|\perp R\rangle \Rightarrow |\perp L\rangle). \tag{3.26}$$

3.5 Actual measurements: Locality or nonlocality?

Here the question relates to actual measurements. Nonlocality is an influence of an $\{\mathcal{L}\}$ measurement at $p_{\{\mathcal{L}\}}$ on an R-state at p_R, with $p_{\{\mathcal{L}\}}$ and p_R being causally separated. In view of eqs. (3.13),(3.16), the influence exists if and only if the measurement is selective. Every actual measurement is selective, so that quantum theory is nonlocal.

We quote Bell [11]: "The paradox of Einstein, Podolsky and Rosen ... was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality..."

It seems pertinent to quote Unruh [2] as well: "... locality is usually used to argue that value that a variable attains must be independent of the choice of experiment carried out in a causally disconnected region (although correlations clearly mean that the value need not be independent of the values obtained for measurements in disconnected regions)."

4 Impossibility of determining causelike order of jumps caused by causally separated measurements

Let (l) and (r) be selective measurements of $\mathcal{L}$ and $\mathcal{R}$ respectively, $p_{(l)}$ and $p_{(r)}$ being causally separated. Let us find a conditional probability $P(l \mid r)$ for $p_{(r)} < p_{(l)}$ and $p_{(l)} < p_{(r)}$. Let N be the number of measurements. We have

$$\text{for} \quad p_{(r)} < p_{(l)}: \quad N = N_r + N_{\bar{r}} = (N_{rl} + N_{r\bar{l}}) + N_{\bar{r}} \tag{4.1}$$

where $\bar{r}$ stands for not r, so that

$$P_{rl}(l \mid r) = \frac{N_{rl}}{N_r} = \frac{N_{rl}/N}{N_r/N} = \frac{P(rl)}{P(r)}; \tag{4.2}$$

$$\text{for} \quad p_{(l)} < p_{(r)}: \quad N = N_l + N_{\bar{l}} = (N_{lr} + N_{l\bar{r}}) + (N_{\bar{l}r} + N_{\bar{l}\bar{r}}) = (N_{lr} + N_{\bar{l}r}) + (N_{l\bar{r}} + N_{\bar{l}\bar{r}}), \tag{4.3}$$

$$P_{lr}(l \mid r) = \frac{N_{lr}}{N_{lr} + N_{\bar{l}r}}. \tag{4.4}$$

Now we have

$$N_{lr} = N_{rl} \quad \text{since} \quad [\mathcal{L}, \mathcal{R}] = 0 \tag{4.5}$$

and

$$N_{lr} + N_{\bar{l}r} = N_r = N_{rl} + N_{r\bar{l}} \quad \text{since} \quad \rho_R^{[\mathcal{L}]} = \rho_R^0. \tag{4.6}$$

Thus

$$P_{lr}(l \mid r) = P_{rl}(l \mid r). \tag{4.7}$$

5 Counterfactuals

5.1 Counterfactual probabilities

Following [1] and the discussion, we consider counterfactuals. Let $\mathcal{R}'$ be an observable for R, such that

$$[\mathcal{R}', \mathcal{R}] \neq 0. \tag{5.1}$$

Let $[\mathcal{L}]$ and (r) measurements be performed, $p_{[\mathcal{L}]}$ and $p_{(r)}$ being causally separated. Our interest here is with the counterfactual probability

$$P^{\mathrm{cf}}(r') \equiv P^{\mathrm{cf}}((r') \mid \{[\mathcal{L}], (r)\}). \tag{5.2}$$

$P^{\mathrm{cf}}(r')$ depends on a state ρ_R in which $\mathcal{R}'$ would be measured and r' would be obtained:

$$P^{\mathrm{cf}}(r') = P(r' \mid \rho_R) = \langle r'| \rho_R |r'\rangle . \tag{5.3}$$

We have

$$\text{for} \quad p_{(r)} < p_{[\mathcal{L}]} : \quad \rho_R = \rho_R^0, \tag{5.4}$$

so that

$$P^{\mathrm{cf}}_{r\mathcal{L}}(r') = \mathrm{Tr}_L \langle r'| \rho^0 |r'\rangle ; \tag{5.5}$$

$$\text{for} \quad p_{[\mathcal{L}]} < p_{(r)} : \quad \rho_R = \sum_l P_{\mathcal{L}r}(l \mid r) \rho_R^{(l)}, \tag{5.6}$$

or, in view of eqs. (4.7) and (3.16),

$$\rho_R = \sum_l P_{r\mathcal{L}}(l \mid r) \frac{\langle l| \rho^0 |l\rangle}{\langle l| \rho_L^0 |l\rangle}. \tag{5.7}$$

Now

$$P_{r\mathcal{L}}(l \mid r) = \frac{P(rl)}{P(r)} = \frac{\langle l| \langle r| \rho^0 |r\rangle |l\rangle}{\langle r| \rho_R^0 |r\rangle}, \tag{5.8}$$

so that

$$P^{\mathrm{cf}}_{\mathcal{L}r}(r') = \sum_l \frac{\langle r| \langle l| \rho^0 |l\rangle |r\rangle \langle r'| \langle l| \rho^0 |l\rangle |r'\rangle}{\langle r| \rho_R^0 |r\rangle \langle l| \rho_L^0 |l\rangle}. \tag{5.9}$$

We emphasize that

$$P^{\mathrm{cf}}_{r\mathcal{L}}(r') \neq P^{\mathrm{cf}}_{\mathcal{L}r}(r'), \tag{5.10}$$

which undermines Stapp's fifth proposition.

5.2 Pure initial state

For ρ^0 given by eq. (3.17), we have

$$P^{\mathrm{cf}}_{r\mathcal{L}}(r') = \mathrm{Tr}_L\{\langle r'| 0\rangle \langle 0| r'\rangle\}, \tag{5.11}$$

$$P^{\mathrm{cf}}_{\mathcal{L}r}(r') = \sum_l \frac{|\ \langle r| \langle l| 0\rangle \langle r'| \langle l| 0\rangle\ |^2}{\mathrm{Tr}_L\{\langle r| 0\rangle \langle 0| r\rangle\}\mathrm{Tr}_R\{\langle l| 0\rangle \langle 0| l\rangle\}}. \tag{5.12}$$

5.3 Two-dimensional Hilbert spaces

For two-dimensional Hilbert spaces of L and R systems, we have

$$\sum_l f(l) = f(l) + f(\bar{l}), \quad \sum_r g(r) = g(r) + g(\bar{r}). \tag{5.13}$$

6 Hardy-type initial state

Following [1] and the discussion, let us consider a Hardy-type initial state (though all is already done by eq. (5.10)):

$$|0\rangle = \frac{\Psi}{\|\Psi\|}, \quad |\Psi\rangle = |l'\rangle\,|r'\rangle - \langle \bar{l} \mid l'\rangle\,\langle r|\,|r'\rangle \mid \bar{l}\rangle\,|r\rangle \tag{6.1}$$

where

$$\mathcal{L} = L2,\ \mathcal{L}' = L1,\ \mathcal{R} = R2,\ \mathcal{R}' = R1; \qquad l = L2+,\ \bar{l} = L2-,\ l' = L1+,\ r = R2+,\ r' = R1-. \tag{6.2}$$

We find

$$\text{for} \quad p_{(r)} < p_{[\mathcal{L}]} : \quad P^{\rm cf}_{r\mathcal{L}}(r') = \frac{|\,\langle l|\, l'\rangle\,|^2 + |\,\langle \bar{l} \mid l'\rangle\,|^2\,(1-|\,\langle r|\, r'\rangle\,|^2)^2}{|\,\langle l|\, l'\rangle\,|^2 + |\,\langle \bar{l} \mid l'\rangle\,|^2\,(1-|\,\langle r \mid r'\rangle\,|^2)} < 1, \tag{6.3}$$

$$\text{for} \quad p_{[\mathcal{L}]} < p_{(r)} : \quad P^{\rm cf}_{\mathcal{L}r}(r') = 1. \tag{6.4}$$

It is impossible to determine which case—(6.3) or (6.4)—takes place. If $p_{(r)} < p_{[\mathcal{L}]}$, the $[\mathcal{L}]$-measurement has nothing to do with $P^{\rm cf}_{r\mathcal{L}}(r')$. If and only if $p_{[\mathcal{L}]} < p_{(r)}$, the (nonselective) $[\mathcal{L}]$-measurement allows for nontrivial statements on counterfactual measurements for R.

7 On the Stapp-Unruh-Mermin discussion

Stapp's fifth proposition [1] is the cornerstone of Stapp's construct. Unruh and Mermin argue that the conclusion of the construct is untenable, but they grant the fifth proposition. Finkelstein [8] argues that the fifth proposition should be specified through consideration of a hypothetical world.

We argue that Stapp's fifth proposition is incorrect. Let us consider that proposition and its proof in the form given by Mermin [3]:

"(I) Whenever the choice of measurement on the left is $L2$, if the measurement on the right is $R2$ and gives $+$, then if $R1$ were instead performed the result would be $-$.

... The validity of this is established by translating it into the language appropriate to the frame of reference in which the events on the left happen first:

(I_L) Whenever the choice of measurement on the left was $L2$, if the measurement done later on the right is $R2$ and gives $+$, then if $R1$ were instead done later on the right, the result would have to be $-$."

In (I_L) a frame is used in which

$$t_{[\mathcal{L}]} < t_{(r)}. \tag{7.1}$$

Had eq. (7.1) implied $p_{[\mathcal{L}]} < p_{(r)}$,

$$t_{[\mathcal{L}]} < t_{(r)} \Rightarrow p_{[\mathcal{L}]} < p_{(r)}, \tag{7.2}$$

(I_L) would have followed from eq. (6.4). But the implication (7.2) is wrong; indeed eq. (7.1) has nothing to do with the problem of nonlocality. It is misleading to invoke reference frames and a time order for causally separated events.

Conclusion

Quantum theory is nonlocal. As for actual measurements, nonlocality manifests itself in and only in selective ones. As for counterfactual nonselective measurements $[\mathcal{L}]$, nonlocality manifests itself if and only if $p_{[\mathcal{L}]} < p_{(r)}$, the fulfillment of which cannot be established in the limits of special relativity.

Quantum jumps and special relativity are incompatible: the former cannot be described in the limits of the latter.

Acknowledgment

I would like to thank Stefan V. Mashkevich for helpful discussions.

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